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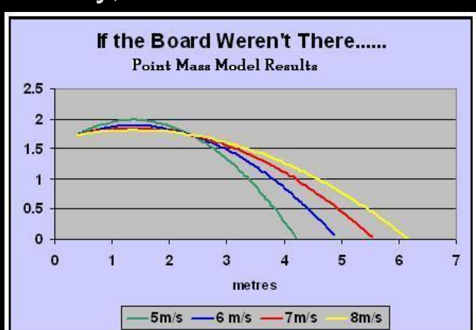
6 Degrees of Freedom

Some phrases from the world of rocket science got pensioned off even before me. Today's young boffins may never exclaim "where's my slide rule?" or "the punch tape's jammed again" or even be aware that "a computer" once meant a person who did trajectory calculations, but the term "6 Degrees of Freedom" is likely to be just as much part of their working life as it was mine.

Before going any further, a warning. There now follows a frankly fairly tedious explanation as to what relevance that has to darts. Any folk not keenly interested in dart ballistics are advised to skip to the slightly more interesting bit at the end!

Hmm, still here? Well, you have been warned! OK, to specify the attitude and position of a projectile, or indeed any object, requires (unless you're Heisenberg) 3 spatial co-ordinates (range, height, and drift, say) and 3 angles (eg up/down, left/right, and axial rotation). Mathematical trajectory models that deal properly with all these are thus called "6 Degree of Freedom" models, or "6DoF" for short.



Luckily, as 6DoF models are a bit messy calculation-wise, for stable projectiles like darts a good approximation to the trajectory can be often gained assuming the projectile always points along its line of flight. This results in a "Point Mass" model (as on the left) where the only aerodynamic force considered is drag. Simpler still is an "In Vacuo" (ie no air and hence no aerodynamics) model where even drag is ignored and gravity is the only force considered. This gives rise to the solution of a parabola.

As a typical dart is heavy for its size and flies comparatively slowly, the effect of the drag force on it is pretty small and the differences between a 6DoF, a Point Mass, and an In Vacuo trajectory are generally only a few millimeters, which is why a dart's trajectory is often, albeit a tad inaccurately, referred to as a parabola.

Because the gravitational acceleration of a body is independent of its mass (at sea level it's just under 10 metres per second squared whether you're an elephant or an ant), the only effect the weight of the dart has on that parabola is through how it's thrown. This is why there's no easy answer to whether heavier darts land higher or lower than lighter ones, differences in how they're thrown will dominate any millimetric extra drag effect on the latter.

That small extra drag effect is covered by Newton’s Second Law of Motion, which can be written F=Ma, or Force equals Mass times Acceleration. Two darts experiencing the same drag force (this ignores differences in size and shape) will therefore decelerate in inverse proportion to their mass

Well done getting this far, only a bit further to go! Newton’s Second Law F=Ma is the basic equation employed in a Point Mass model and there is an equivalent for the rotational motion additionally considered in a 6DoF model. Notation is less standardised here, but I’ll use T=Iq where T is Turning Moment, generally just shortened to Moment, q is angular acceleration and I is Moment of Inertia. T is thus the rotational relative of force and I the rotational relative of mass.

A Moment is a force acting at a distance, like a lever, and the force which stabilises a dart is, as we’ve seen in previous blogs, the Normal Force, which mostly comes from lift from the flights. This acts at a point called the Centre of Pressure (CP), which for darts is thus around the location of the flights.

The distance between a dart’s Centre of Gravity (CG) and CP is called its “Static Margin” and multiplying this by the Normal Force gives what’s contrarily called the “Overturning Moment” (contrarily because it does the opposite of overturning by making the dart point forward, but that is catered for with a minus sign).

So at last to the denouement. Long shafts obviously move the CP back and increase the Static Margin and big flights give more lift, both of which will add correctional oomph to the Overturning Moment. Is that recipe for a more stable dart so simple those who skipped the last eight paras weren’t missing much?

Not so fast! Although the dart will indeed be more stable in terms of Static Margin, in the practical matter of how quickly it corrects in flight there is another factor to consider. Just as Newton’s Second Law says a larger mass M will accelerate more slowly, something with greater Moment of Inertia I will turn more slowly. So if fitting a longer shaft and bigger flights to a dart increases I more than it increases the magnitude of the Overturning Moment, the result will be slower in-flight correction, even though the dart could be regarded as technically more stable.

Maybe, you say, but shafts and flights are light compared to a barrel, perhaps only around 10% of a dart’s total weight. If a dart’s Moment of Inertia is somehow related to its mass, surely tweaking the set-up can’t affect it that much?

And that, my friends, is a topic for the future.